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# WEIGHTED EXPONENTIAL TRICHOTOMY OF DIFFERENCE EQUATIONS AND ASYMPTOTIC BEHAVIOR FOR NONLINEAR SYSTEMS

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**Abstract.** Using the notion of weighted exponential trichotomy for difference equations we prove the existence of bounded solutions and solutions that go to zero at the infinity in the future and the past for nonlinear systems. The results are consequence of applications of different point fixed theorems, namely, the Contraction Principle, the Schauder-Tychonoff and the Leray-Schauder fixed point technique.
**Key words and phrases:** Difference equation; Asymptotic behavior;
Weighted exponential trichotomy; Nonlinear Difference Equations.

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# 1 Introduction

In this paper we consider the linear system of difference equations

$$x(n+1) = A(n)x(n), \quad n \in \mathbb{Z}, \tag{1}$$

where $A(n)$ is a $m \times m$ invertible matrix defined on $\mathbb{Z}$, and its perturbation,

$$x(n+1) = A(n)x(n) + f(n, x(n)), \quad n \in \mathbb{Z}, \tag{2}$$

i.e., the nonlinear difference system with $f : \mathbb{Z} \times \mathbb{R}^m \to \mathbb{R}^m$. Recently in [17] and [3] the concept of *weighted exponential trichotomy for difference equations* was introduced, which is a generalization of the exponential trichotomy concept. Now, we proceed to remember this important definition.

**Definition 1.1** *Let $X(n)$ denote the fundamental matrix of equation (1) with $X(0) = I$. Suppose that there are $h$ and $k$ two positive sequences $\{h(t)\}_{t\in\mathbb{Z}}$, $\{k(t)\}_{t\in\mathbb{Z}}$; three mutually orthogonal $m \times m$ matrix projections*

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