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Switching control of linear systems for generating chaos ☆

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Abstract

In this paper, a new switching method is developed, which can be applied to generating different types of chaos or chaos-like dynamics from two or more linear systems. A numerical simulation is given to illustrate the generated chaotic dynamic behavior of the systems with some variable parameters. Finally, a circuit is built to realize various chaotic dynamical behaviors.

1. Introduction

Chaotic systems have been extensively investigated in the past decades and numerous interesting results have appeared in the literature, see [1–18] and references therein. Recently, chaos has been found to be very useful in a variety of applications such as secure communications, signal processing, industrial electronics, power systems, and biomedical engineering. This provides a strong motivation for the current research on exploiting new chaotic systems and their implementations. Some new chaotic systems have been developed in [14–18], but there does not seem to have a general methodology for generating chaos. In this paper, we propose a new switching method for two linear systems. It is shown that chaos can be generated by applying an appropriate switching rule. This new switching rule can generate different types of chaos or chaos-like behaviors from different pairs of linear systems.

The rest of the paper is organized as follows. In Section 2, we describe the switching rule for two general linear systems. In Section 3, we analyze the switched system for a set of parameters and show how chaotic dynamics are generated. Numerical simulations are given in Section 4 and a circuit is built in Section 5 for the realization of a chaotic system by means of switching control. Finally, some concluding remarks are given in Section 6.

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2. The new chaotic generator

Consider the following linear systems

$$\dot{X} = A_1X + b_1, \tag{1}$$

$$\dot{X} = A_2X + b_2, \tag{2}$$

where X is an n-dimensional state vector, A_1, A_2 are $n \times n$ constant matrices, and b_1, b_2 are n-dimensional constant vectors.

Assume that system (1) has an unstable equilibrium X_1^*, and system (2) has a stable equilibrium X_2^*. Let

$$X_0^* = 1/2(X_1^* + X_2^*), \quad \text{and} \quad l = 1/2\|X_1^* - X_2^*\|.$$

Let us define the following three regions:

$$\Sigma_1 = \{X \mid \|X - X_0^*\| \leqslant k\},$$
$$\Sigma_2 = \{X \mid k < \|X - X_0^*\| < m\},$$
$$\Sigma_3 = \{X \mid \|X - X_0^*\| \geqslant m\},$$

where k and m are such that $l < k < m < +\infty$.

The switching rule is constructed as follows. When system (1) is active, it will switch to system (2) at time t_1 if $X(t_1) \in \Sigma_3$. Similarly, when system (2) is active, it will switch to system (1) at time t_2 if $X(t_2) \in \Sigma_1$.

With this switching rule, the switched system will generate chaos or chaos-like behavior if the system parameters are properly chosen. The details are discussed and demonstrated in the next section.

3. Dynamical analysis of the switched system

We consider the switched systems (1) and (2), under the switching rule described in the previous section with the following specifications:

$$A_1 = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & c \end{pmatrix}, \quad b_1 = \begin{pmatrix} 0 \\ 0 \\ d \end{pmatrix}, \tag{3}$$

$$A_2 = \begin{pmatrix} f & 0 & 0 \\ 0 & g & h \\ 0 & -h & g \end{pmatrix}, \quad b_2 = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}, \tag{4}$$

where we assume that $a > 0$, $c \neq 0$, $f < 0$, $g < 0$, and the constants k and m are such that $-d/(2c) < k < m$. System (1) with A_1 and b_1 given by (3) will be referred to as system (3). Similarly, system (2) with A_2 and b_2 given by (4) will be referred to as system (4).

By solving $\dot{X} = 0$, the equilibrium of system (3) is found to be $X_1^* = (0, 0, -d/c)$. The Jacobian matrix J at this point is

$$J = \begin{pmatrix} a & b & 0 \\ -b & a & 0 \\ 0 & 0 & c \end{pmatrix},$$

whose eigenvalues are $\lambda_{1,2} = a \pm bi$, and $\lambda_3 = c$. Therefore, the equilibrium $(0, 0, -d/c)$ is unstable.

The equilibrium of system (4) is $X_2^* = (0, 0, 0)$. The eigenvalues of A_2 are $\lambda_{1,2} = g \pm hi$, and $\lambda_3 = f$. Thus, the equilibrium $(0, 0, 0)$ is stable. Therefore,

$$X_1^* = (0, 0, -d/c), \quad X_2^* = (0, 0, 0),$$
$$X_0^* = (0, 0, -d/(2c)), \quad l = |d/(2c)|, \quad l < k < m,$$
$$\Sigma_1 = \left\{X \mid \sqrt{X_1^2 + X_2^2 + (X_3 - l)^2} \leqslant k\right\},$$
$$\Sigma_2 = \left\{X \mid k < \sqrt{X_1^2 + X_2^2 + (X_3 - l)^2} < m\right\},$$
$$\Sigma_3 = \left\{X \mid \sqrt{X_1^2 + X_2^2 + (X_3 - l)^2} \geqslant m\right\}.$$

When system (3) is active, we define

$$V_1 = X_1^2 + X_2^2, \quad \text{and} \quad W_1 = \sqrt{X_1^2 + X_2^2 + (X_3 - l)^2}.$$

Then,

$$\dot{V}_1 = 2X_1\dot{X}_1 + 2X_2\dot{X}_2 = 2aX_1^2 + 2aX_2^2 = 2aV_1.$$

Thus, $V_1 = V_1(0)e^{2at}$. That is, $V_1 \to +\infty$ and $W_1 \to +\infty$, as $t \to +\infty$. Hence, there exists $t = T_1$ such that $W_1(T_1) \geqslant m$, and $X \in \Sigma_3$. We then apply the switching rule to switch system (3) to system (4).

When system (4) is active, we have

$$\dot{X}_1 = fX_1.$$

Thus,

$$X_1 = X_1(0)e^{ft},$$

and hence $X_1 \to 0$, as $t \to +\infty$.

Let $V_2 = X_2^2 + X_3^2$ and $W_2 = \sqrt{X_1^2 + X_2^2 + (X_3 - l)^2}$. Then,

$$\dot{V}_2 = 2X_2\dot{X}_2 + 2X_3\dot{X}_3 = 2gX_2^2 + 2gX_3^2 = 2gV_2.$$

Thus, $V_2 = V_2(0)e^{2gt}$. That is, $V \to 0$, and $X_2, X_3 \to 0$, as $t \to +\infty$.

Therefore, $W_2 \to l$, as $t \to +\infty$. Hence at some $t = T_2$, we have $W_2(T_2) \leqslant k$ and $X \in \Sigma_1$, and the switching rule is then applied to switch system (4) to system (3).

From the foregoing discussion, we observe that as $t \to +\infty$, the system switches between system (3) and system (4), and the state orbits go out from or into the region Σ_2 repeatedly. Hence, the state orbits are folded and stretched repeatedly, leading to the generation of chaos or chaos-like behaviors.

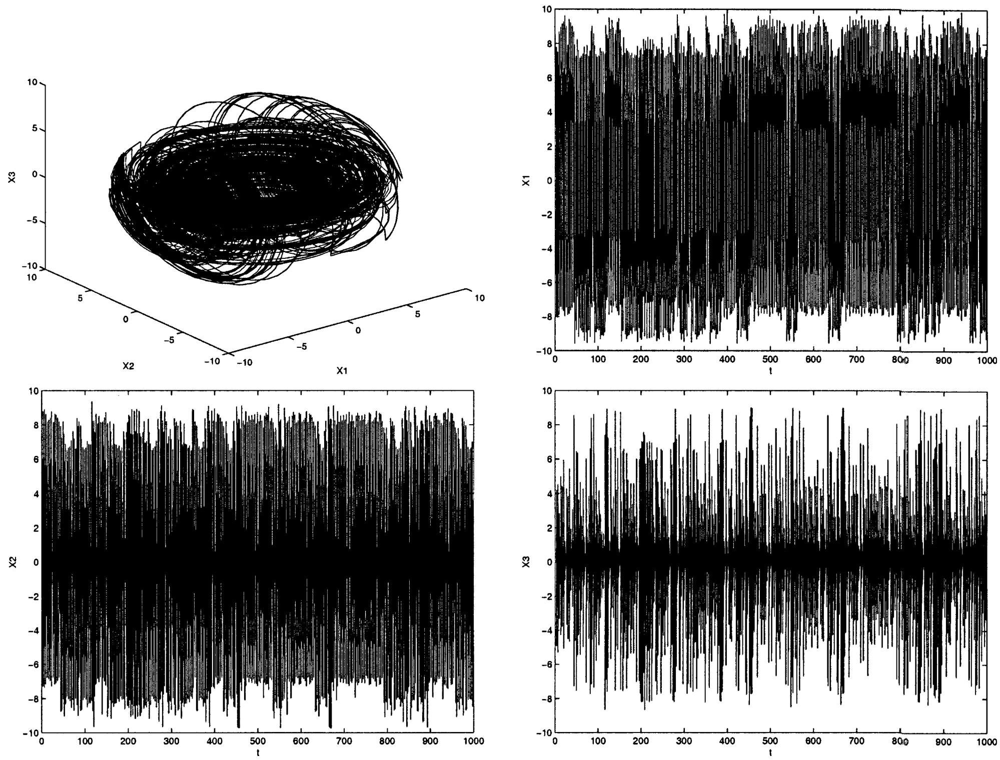

Fig. 1. Phase portraits and state orbits of the switched system, with $a = 0.9$, $b = 15$, $c = 0.5$, $d = 1.0$, $f = -0.5$, $g = -1$, $h = 20$, $k = 3$, and $m = 10$, starting from the initial point $(10, 0, 0)$.

4. Dynamical behavior and numerical simulation

In this section, we shall investigate the dynamical behaviors of the switched systems (3) and (4) through some numerical simulations. Consider the switched system (3) and (4), with $a = 0.9$, $b = 15$, $c = 0.5$, $d = 1.0$, $f = -0.5$, $g = -1$,

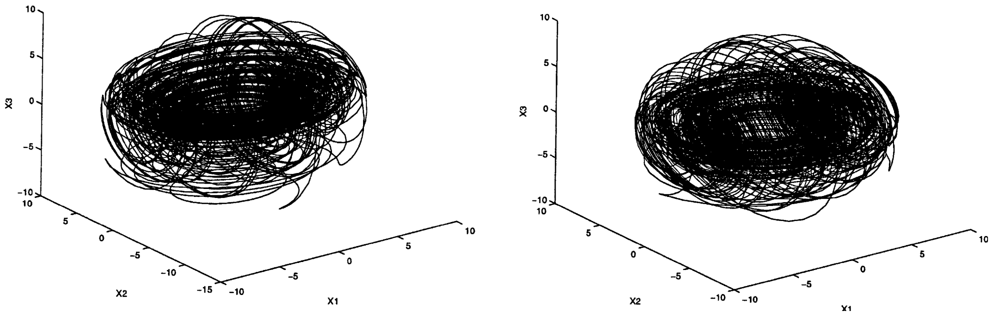

Fig. 2. Phase portraits of the system, with $a = 0.5$ and $a = 2$, respectively, both starting from the initial point $(10, 0, 0)$.

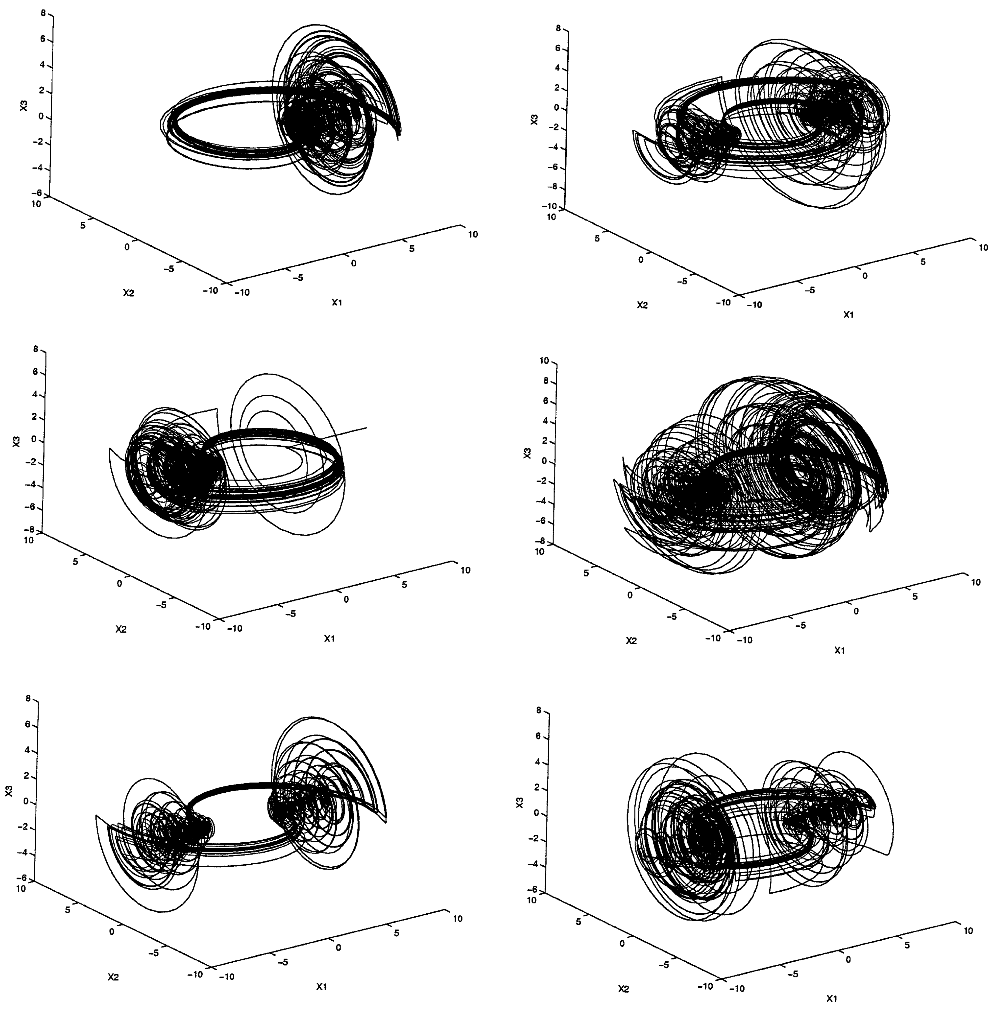

Fig. 3. Phase portraits of the system with, $a = 2.6$, $a = 2.83$, $a = 3$, $a = 4.2$, $a = 5$, and $a = 6$, respectively, all starting from the initial point $(10, 0, 0)$.

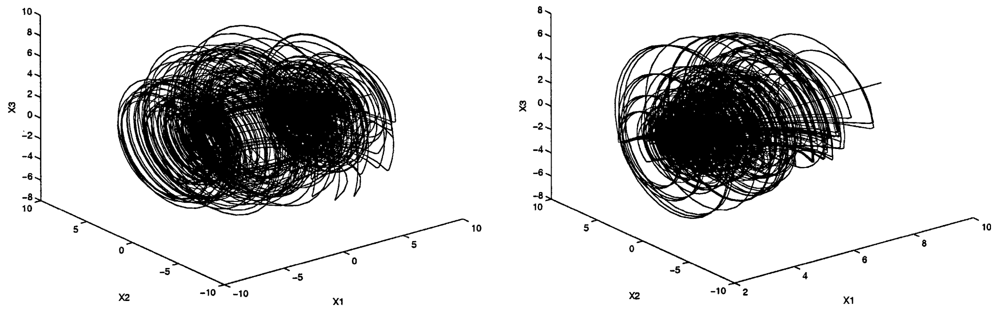

Fig. 4. Phase portraits of the system, with $a = 7$ and $a = 15$, respectively, both starting from the initial point (10,0,0).

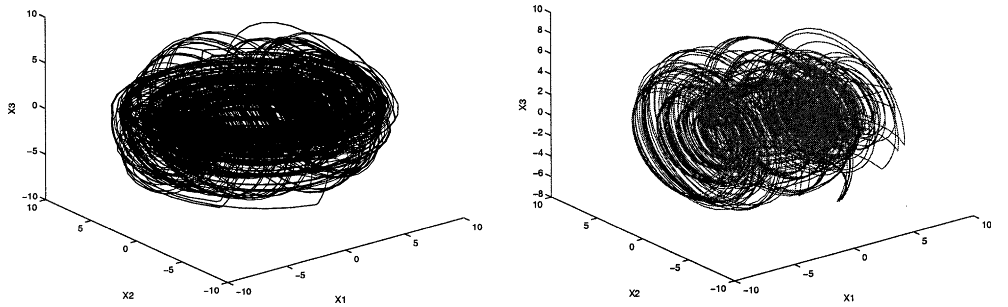

Fig. 5. Phase portraits of the system, with $b = -15$ and $b = 2$, respectively, both starting from the initial point (10,0,0).

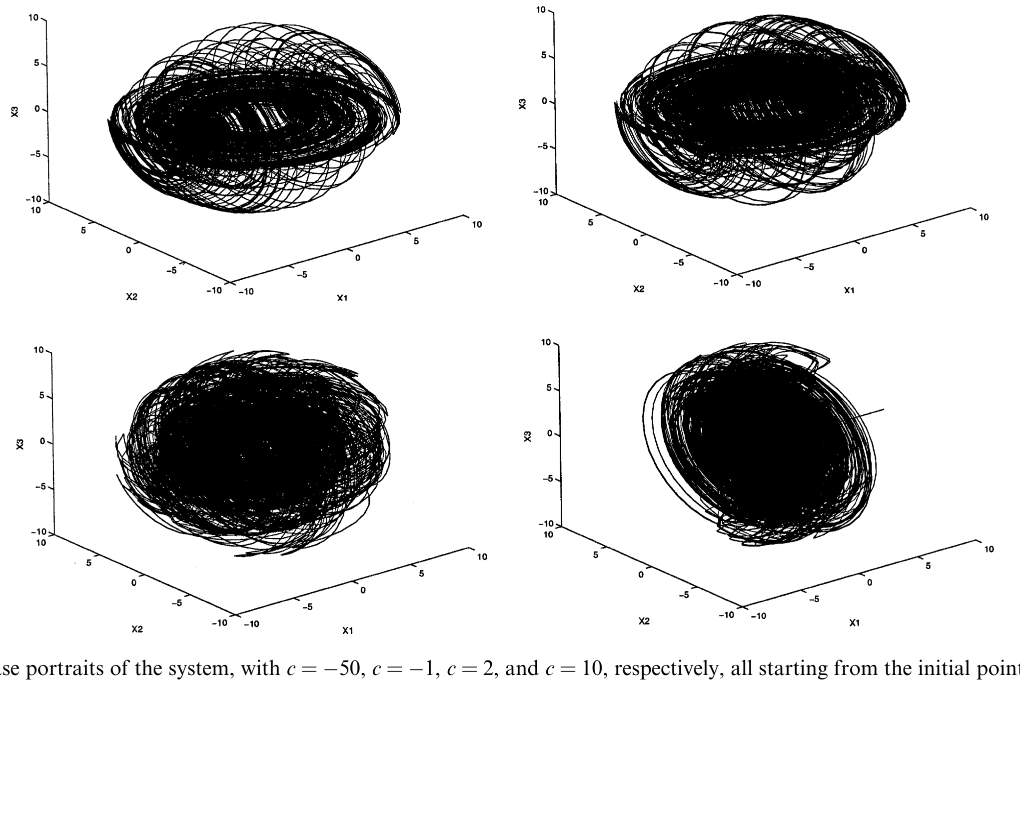

Fig. 6. Phase portraits of the system, with $c = -50$, $c = -1$, $c = 2$, and $c = 10$, respectively, all starting from the initial point (10,0,0).

$h = 20$, $k = 3$, and $m = 10$. The system generates chaotic attractors, and the maximum Lyapunov exponent of this attractor is $LE = 1.8306$. The phase portraits and state orbits of the system are depicted in Fig. 1.

4.1. Variation of parameter *a*

Let a be varied while holding other parameters fixed. Then, the corresponding dynamical behaviors of the switched system are as described below:

1. When $0 < a < 2.5$, the system generates chaos, as shown in Fig. 2.
2. When $2.5 < a < 6.1$, the state of the system converges to periodic or period-like orbits, as shown in Fig. 3.
3. When $a > 6.1$, the system generates various chaos or chaos-like orbits, as shown in Fig. 4.

4.2. Variation of parameter *b*

Let b be varied while holding other parameters fixed. For almost all b, the system generates chaos or chaos-like orbits, as shown in Fig. 5.

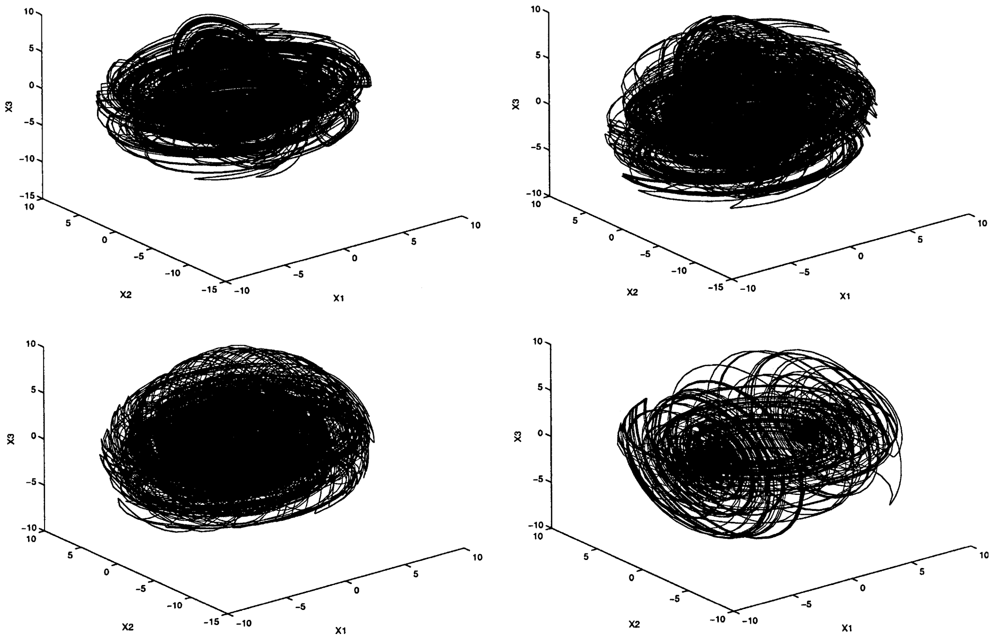

Fig. 7. Phase portraits of the system, with $f = -50$, $f = -10$, $f = -2$, and $f = -0.1$, respectively, all starting from the initial point $(10, 0, 0)$.

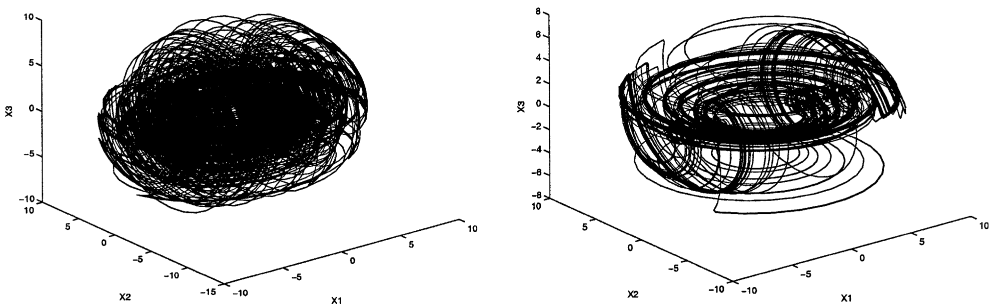

Fig. 8. Phase portraits of the system, with $g = -0.5$ and $g = -5$, respectively, both starting from the initial point $(10, 0, 0)$.

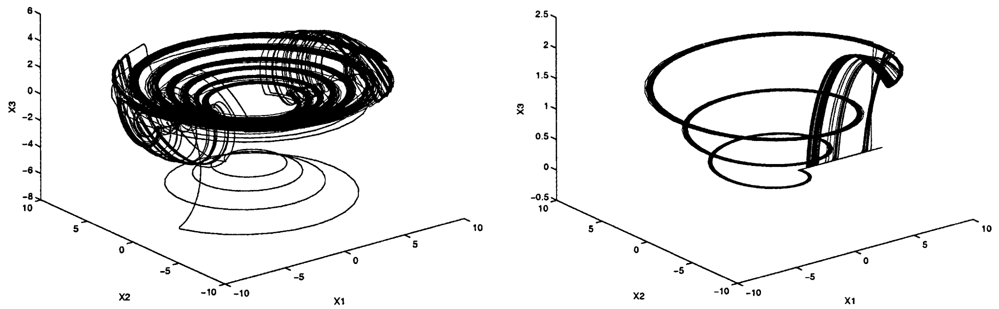

Fig. 9. Phase portraits of the system, with $g = -10$ and $g = -50$, respectively, both starting from the initial point (10,0,0).

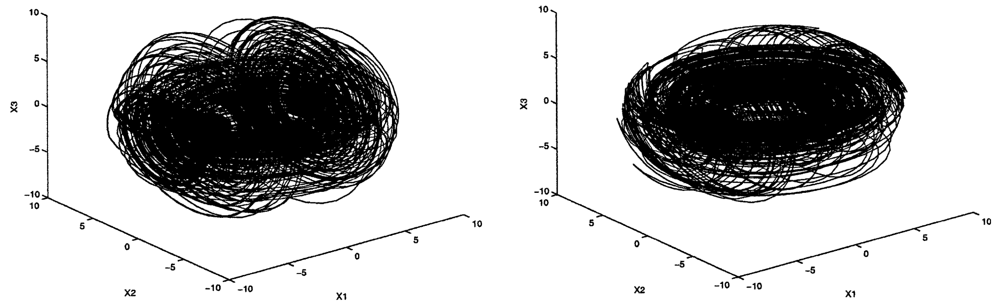

Fig. 10. Phase portraits of the system, with $h = -50$ and $h = 5$, respectively, both starting from the initial point (10,0,0).

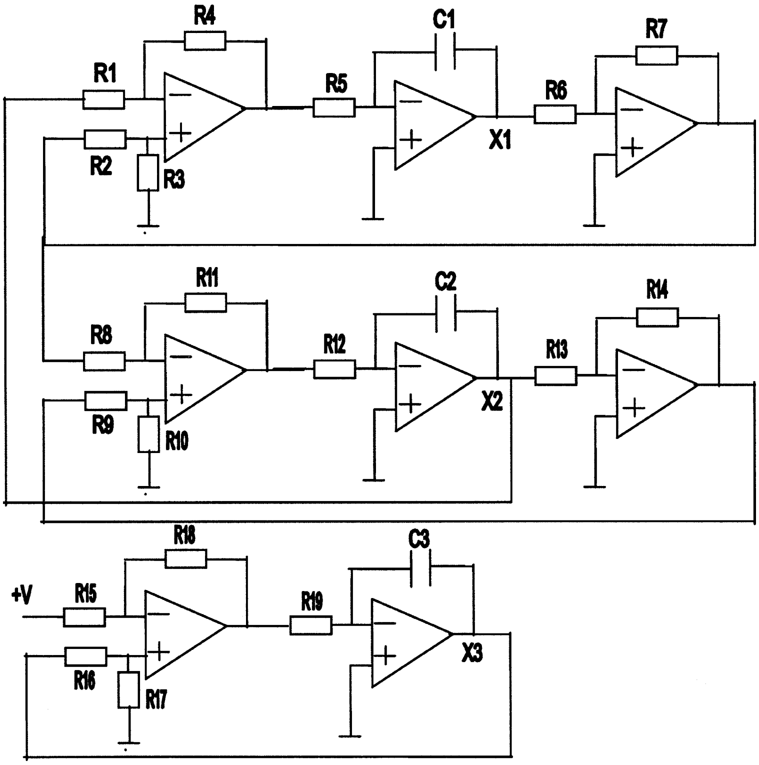

Fig. 11. Circuitry realization of system (3).

4.3. Variation of parameter c

Vary c while holding other parameters fixed so as to ensure that $|d/(2*c)| < k$, i.e., $|c| > 1/6$. The dynamical behavior of the switched system is as shown in Fig. 6. For almost all c, the system generates chaos or chaos-like behavior.

4.4. Variation of parameter f

Vary f within $(-\infty, 0)$ while holding other parameters fixed. The dynamical behaviors of the switched system are as shown in Fig. 7. We observe that the system is always chaotic for all values of f.

4.5. Variation of parameter g

Vary g within $(-\infty, 0)$ while holding other parameters fixed. The dynamical behaviors of the switched system are as follows:

1. When $-5 < g < 0$, the system generates chaos or chaos-like behavior, as shown in Fig. 8.
2. When $g < -5$, the system is periodic, as shown in Fig. 9.

4.6. Variation of parameter h

Vary h while holding other parameters fixed. Numerical simulations show that the system is chaotic or chaos-like for almost all values of h, as shown in Fig. 10.

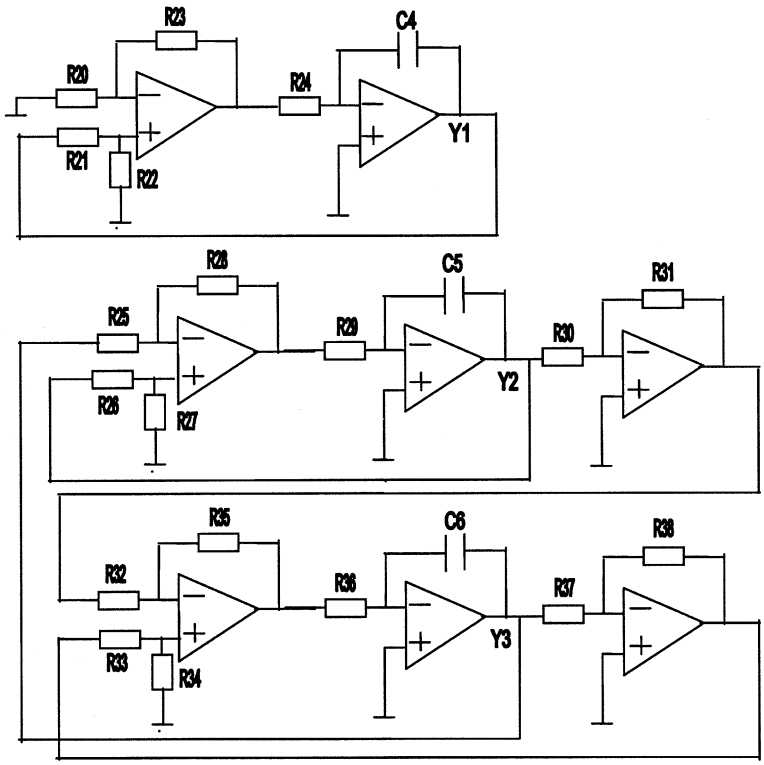

Fig. 12. Circuitry realization of system (4).

5. Circuitry realization

In this section, a circuit is designed to realize the chaotic systems, where the system parameters are chosen as follows: $a > 0$, $b > 0$, $c < 0$, $d > 0$, $f < 0$, $g < 0$, and $h > 0$. The circuit of system (3) is shown in Fig. 11, while the circuit of system (4) is shown in Fig. 12.

6. Conclusions

This paper has presented a new switching control method for generating chaos or chaos-like dynamics. This is achieved by applying a simple switching rule to two linear systems. A special feature is that this switched system can be realized easily by electric circuits, thus it has potential applications in engineering and technology as a chaos generator. Furthermore, this method can be extended to situations involving three or more linear systems for generating chaotic attractors with more complicated topologies.

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